Write the **formal definition** of a function used in discrete math. **Use correct English and complete sentences. You may use symbolic logic and set notation shown in lecture, if you use it correctly.** **SCORE: ____ / 10 PTS**

(1) A FUNCTION F FROM SET A TO SET B, (1)
(1) IS A RELATION WITH DOMAIN A AND CO-DOMAIN B (1) SUCH THAT

$\forall x \in A, \exists y \in B : (x,y) \in F$, (3)

AND $\forall x \in A, \forall y,z \in B, (x,y) \in F \text{ AND } (x,z) \in F \rightarrow y = z$ (3)

Determine if $((p \oplus \sim q) \leftrightarrow \sim r) \rightarrow (q \vee r)$ is a tautology, a contradiction or neither. NEITHER **SCORE: ____ / 15 PTS**

State your conclusion clearly. (3) EACH

p	q	r	$\sim q$	$p \oplus \sim q$	$\sim r$	$(p \oplus \sim q) \leftrightarrow \sim r$	$q \vee r$	
T	T	T	F	T	F	F	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	F	F	F	T	T	T
F	T	F	F	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	T	T	T	F

Use the predicate $P(x, y) =$ "$x^3 = y$" to write the following sentences symbolically. **SCORE: ____ / 15 PTS**

NOTE: Your answers must involve *P*, and must NOT contain any English words nor the cube root function.

[a] "w is a perfect cube" (where w is a particular real number) "THERE IS AN INTEGER X SUCH THAT $x^3 = w$"

$\exists x \in \mathbb{Z} : P(x, w)$

(2½) EACH

[b] "8 times a perfect cube is also a perfect cube"

$\forall w \in \mathbb{Z}, (\exists x \in \mathbb{Z} : P(x,w) \rightarrow \exists y \in \mathbb{Z} : P(y, 8w))$

"FOR ALL INTEGERS W, IF W IS A PERFECT CUBE THEN 8W IS A PERFECT CUBE"

Let $A = \{-2, 0, 3, 4\}$ and $B = \{2, 5, 6\}$.

SCORE: ____ / 15 PTS

Let R be the relation from A to B such that aRb if and only if 5 is a factor of $a + 2b$.

a \ b	2	5	6
	4	10	12
−2	2	8	(10)
0	4	(10)	12
3	7	13	(15)
4	8	14	16

[a] Write R in set-roster notation.

$\{(-2,6), (0,5), (3,6)\}$ (2½ EACH)

(3½) NOTATION

[b] Is R a function ? **Justify your answer briefly.**

(4) NO. 4 ISN'T RELATED TO ANYTHING

Prove that the following argument is valid using the Rules of Inference.

SCORE: ____ / 20 PTS

Give the reason for each step as shown in lecture. **Do NOT rewrite any of the hypotheses using logical equivalences.**

① $y \to r$
② $w \vee \sim s$
③ $\sim q \to \sim r$
④ $\sim s \to y$
⑤ $w \to r$
$\therefore q$

$\sim S \to y$ ④ (1½)
$y \to r$ ① (1½)
⑥ $\therefore \sim S \to r$ TRAN ④

$W \vee \sim S$ ② (1½)
$W \to r$ ⑤ (1½)
$\sim S \to r$ ⑥
⑦ $\therefore r$ CASE (4½)

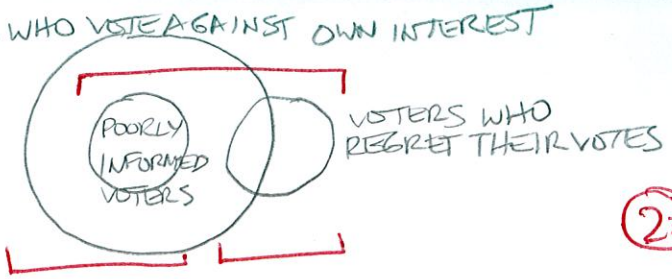
$\sim q \to \sim r$ ③ (1½)
r ⑦
$\therefore q$ MT ④

Indicate whether the following argument is valid or invalid.

SCORE: ____ / 10 PTS

Support your answer using a clearly labelled diagram.

All poorly informed voters vote against their own interest.
Some people who vote against their own interest regret their votes.
Therefore, some poorly informed voters regret their votes.



INVALID

(2½ EACH)

Fill in the blanks. **NOTE: Your answers must NOT contain any English words.** **SCORE: ____ / 20 PTS**

[a] The disjunction of r and s is written symbolically as $r \vee s$. (2)

[b] The symbolic definition of $P \times M$ is $\{(x,y) \mid x \in P \wedge y \in M\}$. (3)

[c] The symbolic definition of $C \subseteq A$ is $\forall x \in C,\ x \in A$. (3)

[d] The set $\{\{\{a\}\},\{\{a\},a\},\{a\},a\}$ contains 4 (2) elements. (NOTE: The answer is a number.)

[e] If $P(x) =$ "x is even" and $Q(x) =$ "x is a prime number", then

$\{x \in \mathbf{Z}^+ \mid P(x) \wedge \sim Q(x)\} = \{4, 6, 8, 10, \ldots\}$ (3). (NOTE: The answer must be in set-roster notation.)

[f] $\{s,o\} \times \{b,a,d\} = \{(s,b),(s,a),(s,d),(o,b),(o,a),(o,d)\}$ (5)

Consider the statement "An error message is generated unless the quiet flag is specified". **SCORE: ____ / 10 PTS**

NOTE: Your answers must be in complete sentences, and must NOT contain any symbols nor variables.

(2½ EACH)

[a] Write a logically equivalent statement using "if" / "then".

IF THE QUIET FLAG IS NOT SPECIFIED, THEN AN ERROR MESSAGE IS GENERATED

[b] Write a logically equivalent statement using "is necessary for".

AN ERROR MESSAGE BEING GENERATED IS NECESSARY FOR THE QUIET FLAG NOT BEING SPECIFIED.

[c] Write a logically equivalent statement using "only if".

THE QUIET FLAG WAS NOT SPECIFIED ONLY IF AN ERROR MESSAGE IS GENERATED

[d] Write the negation of the statement.

THE QUIET FLAG IS NOT SPECIFIED AND AN ERROR MESSAGE IS NOT GENERATED

Fill in the blanks. **NOTE: Your answers must NOT contain any symbols nor variables.** **SCORE: ____ / 15 PTS**

[a] Given the conditional $q \rightarrow p$, p is called the CONCLUSION, q is called the HYPOTHESIS,

$p \rightarrow q$ is called the CONVERSE, and $\sim p \wedge q$ is called the NEGATION.

[b] The argument "If today is Thursday, then Math 22 meets today. Today is not Thursday. Therefore, Math 22 does not meet today."

$p \rightarrow q$
$\sim p$
$\therefore \sim q$

(3 EACH)

is an example of INVERSE ERROR.

The following table shows the cargo that was delivered by 3 drivers during 3 months. **SCORE: ____ / 20 PTS**

	June	July	August
Alan	Appliances	Machinery	No delivery
Barb	Furniture	Machinery	No delivery
Carl	No delivery	Appliances	Furniture

Let D = set of drivers = {Alan, Barb, Carl}.
Let M = set of months = {June, July, August}.
Let C = set of cargo = {appliances, furniture, machinery}.

(2½) EACH

Determine if the following statements are true or false.
Justify your answer as shown in lecture. Use as few examples/counterexamples as possible.

[a] $\exists d \in D : \forall m \in M, d$ made a delivery during m

FALSE

FOR EACH $d \in D$
TEST $\forall m \in M$, d MADE A DELIVERY DURING m

d = ALAN $\forall m \in M$, ALAN MADE A DELIVERY DURING m — FALSE, m = AUGUST

d = BARB — BARB — FALSE, m = AUGUST

d = CARL — CARL — FALSE, m = JUNE

[b] $\forall c \in C, \exists d \in D : d$ delivered c

TRUE

FOR EACH $c \in C$
TEST $\exists d \in D$: d DELIVERED c

c = APPLIANCES $\exists d \in D$: d DELIVERED APPLIANCES — TRUE, d = ALAN OR CARL

c = FURNITURE — FURNITURE — TRUE, d = BARB OR CARL

c = MACHINERY — MACHINERY — TRUE, d = ALAN OR BARB